# **SOFR Curve Construction**

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## SOFR Curve Construction

I have built the SOFR curve as of closing 8/25/2023. The accuracy to Bloomberg benchmark is always within 0.02bp. The library is written in Python, using datasets from Bloomberg.

See below the overnight forward curve and the par swap yield curve.

A graph with lines and numbers

Description automatically generated

A graph with a line

Description automatically generated

### Assumptions

My focus in on the overnight forward curve backboned upon SOFR Index, which can easily transfer to par swap rate curve. The initial date is set to be 8/25/2023, settlement is 8/27/2023.

* Using the SR3 futures in the front end (<3yrs), adjusted by convexity.
* Using Swap rates for 3+ tenor, first layer is 5y, 7y, 10y, 15y, 20y, 30y, 40y and 50y. and implemented the skewness adjustment for 4y, 6y, 8y, 9y, 12y, 25y, 35y and 45y, which are the second layer. In practice, I would implement 3 layers:
  + First layer: Cash yield on (5y, 10y, 30y) + corresponding swap spreads
  + Second layer: 4y, 7y, 15y, 20y
  + Third layer: 6y, 8y, 9y, 11y, 12y, 25y, 40y, 50y
* Using Log-linear interpolation on DF for front end, and log-cubic spline interpolation for 3+ tenor.
* The curve is solved by auto-differentiation under Dual number. Levenberg-Marquardt method is used to blend both the gradient descent and Gauss-Newton method.
* No turns, no holidays and no business day convention for simplicity.
* Knots are placed at strategic points as IMM dates, rather than at the maturity of par tenor. instruments. This is to avoid large exchange basis and carry fluctuation when market advanced.

### Market Instruments



### Front End Curve

Here we zoomed in to have a better view on the front end curve. For up to 3y, we have

A graph with a line going up

Description automatically generated

The nodes are placed at FOMC meeting dates.

Note that the future convexity adjustment is based on the following 2 papers.

* [Hull. 2002. Options, Future and Other Derivatives](https://rads.stackoverflow.com/amzn/click/com/B01K94T4CG) p. 566.
* [Piterbarg and Renedo. 2006. Eurodollar Futures Convexity Adjustments in Stochastic Volalitiy Model. 2006](https://www.risk.net/journal-of-computational-finance/2160379/eurodollar-futures-convexity-adjustments-in-stochastic-volatility-models)

The mean reversion speed is set to be 0.03 and rate vol is set to be 140bps. The closing for SFRM5 is 96.075, so if you look at *SFRM5C 96.0000 COMB Comdty,* this serial option has implied volatility at 37.188% which at current rates (3.925%) is around 140bps.

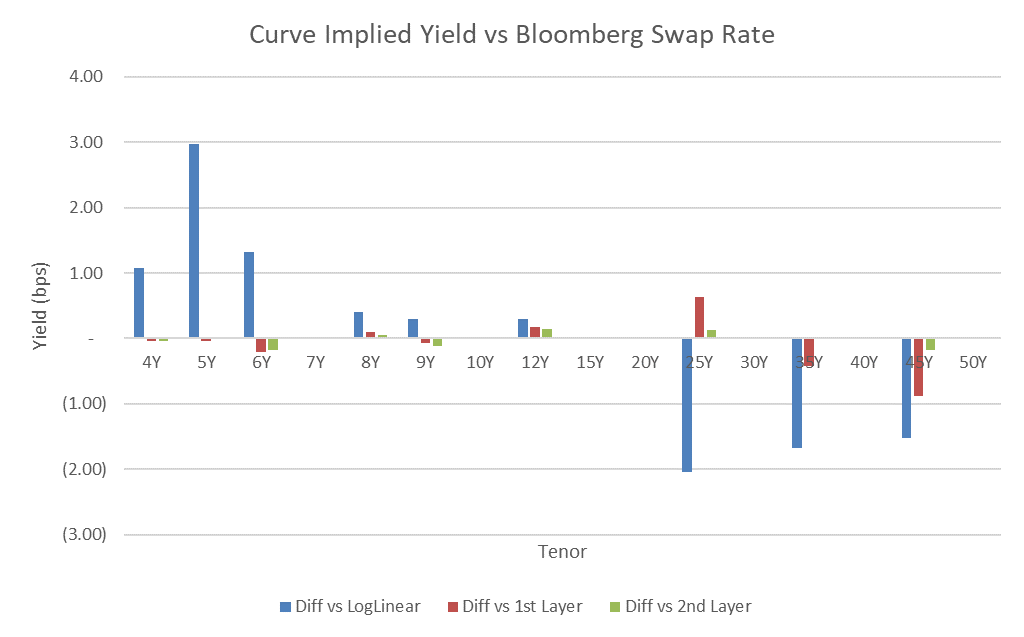
|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Start Date** | **Days** | **Ticker** | **Last** | **Implied Forward** |
| 08/30/2023 | 21 | SFRM3 | 94.6847 | 5.3153 |
| 09/20/2023 | 91 | SFRU3 | 94.545 | 5.4543 |
| 12/20/2023 | 91 | SFRZ3 | 94.51 | 5.4868 |
| 03/20/2024 | 91 | SFRH4 | 94.645 | 5.3482 |
| 06/19/2024 | 91 | SFRM4 | 94.925 | 5.0634 |
| 09/18/2024 | 91 | SFRU4 | 95.295 | 4.6874 |
| 12/18/2024 | 91 | SFRZ4 | 95.645 | 4.3304 |
| 03/19/2025 | 91 | SFRH5 | 95.91 | 4.0572 |
| 06/18/2025 | 91 | SFRM5 | 96.075 | 3.883 |
| 09/17/2025 | 91 | SFRU5 | 96.16 | 3.7877 |
| 12/17/2025 | 91 | SFRZ5 | 96.195 | 3.7414 |
| 03/18/2026 | 91 | SFRH6 | 96.215 | 3.7092 |
| 06/17/2026 | 91 | SFRM6 | 96.235 | 3.6759 |

### 3y+ Curve Residual control

In the daily trading activities, we need to mark level 2 and level 3 residuals according to broker’s screen. I have also implemented this in my curve, which gives me control to skew the curve on 4y, 6y, 8y, 9y, 12y, 25y, 35y and 45y. this is implemented by having a layered curve set.

|  |  |  |
| --- | --- | --- |
| **Input Instruments** | **First Layer Rate (bps)** | **Second Layer Skew (bps)** |
| 4y |  | *0* |
| 5y | 421.5 |  |
| *6y* |  | *0* |
| 7y | 406.3 |  |
| *8y* |  | *-0.05* |
| *9y* |  | *-0.05* |
| 10y | 396.75 |  |
| *12y* |  | *-0.02* |
| 15y | 393.125 |  |
| 20y | 386.525 |  |
| *25y* |  | *-0.5* |
| 30y | 362.7 |  |
| *35y* |  | *0.4* |
| 40y | 340.845 |  |
| *45y* |  | *0.7* |
| 50y | 319.735 |  |

By implement the skew, our final curve (i.e., the 2nd layer below) have less than 0.02bps difference to Bloomberg’s closing yield.



## 